Math 53: Multivariable Calculus **Sections 201, 204**

Worksheet for 2021-10-11

Conceptual questions

Question 1. Let R be the rectangle with corners $(0,0), (2,0), (2,3), (0,3)$. Suppose that $f(x,y)$ is a function such that $f_x > 0$ and $f_y < 0$ on R. If we want to use a Riemann sum to *overestimate* the integral $\iint_R f(x,y)\,\mathrm{d}x\,\mathrm{d}y$, where we break up R into six 1×1 squares, where should we pick the sample points?

Question 2. You are asked to integrate some function $f(x,y)$ over some region R in the plane. In each of the following scenarios, say whether you are inclined to use the integration order $\mathrm{d}x\,\mathrm{d}y$ or the integration order $\mathrm{d}y\,\mathrm{d}x$. How firm is your conviction in each case (i.e. is the other order potentially still worth trying)?

(a) The function is $f(x,y) = e^{x^2}$.
(b) The region R is the triangle with corners $(0,0), (1,1), (1,-1)$.
(c) The region R is the region defined by $x^2 \le y \le x$.
(d) The region R is the bounded region between the two curves $x = y - y^3$ and $x = y^2 - 1$.

Question 3. What region R maximizes the value of the integral below?

$$\iint_R (3 - x^2 + 2x - 4y^2)\,\mathrm{d}x\,\mathrm{d}y$$

Question 4.

(a) (Warm-up) Evaluate the integral

$$\int_{-3}^{3} \arctan(x^3)\,\mathrm{d}x.$$

Hint: What happens if you apply the change of variables $u = -x$?

(b) Evaluate the integral

$$\int_{-1}^{1} \int_{1-\sqrt{4-4y^2}}^{1+\sqrt{4-4y^2}} e^{x^2+y^2} \sin y\,\mathrm{d}x\,\mathrm{d}y.$$

Hint: Switch the order of integration.